Modelling and Experimental Characterisation of Hygrothermoelastic Stress in Polymer Matrix Composites

Marco Gigliotti,[1] *Frédéric Jacquemin,*[2] *Jérôme Molimard,*[3] *Alain Vautrin*[*3]

Summary: The present paper is concerned with the modelling and the simulation of hygrothermal deformation of composite laminates. The temperature and moisture fields are established by employing the Fick's law for transient and cyclical environmental conditions, then the Classical Plate Theory (CLT) adapted for taking into account such conditions is applied. The hygrothermoelastic law of the composite is supposed to be constant but the diffusion coefficients depend on the temperature. The paper shows the ability of the model to handle complex environmental loading, close to service conditions. Finally, a model of plate with moderate rotations is introduced to predict the nonlinear deformations of unsymmetric plates under temperature and moisture cycling conditions.

Keywords: cyclical hygrothermal load; hygrothermal stress fields; nonlinear deformations of composite plates; temperature and moisture fields; transient hygrothermal load

Introduction

The need for weight saving and design flexibility indicates polymer based composite materials (also known as fibre reinforced plastics, FRP) as primary candidates for novel engineering systems. To make composite materials competitive with other well controlled materials (alloys, for instance) in automotive and aeronautical applications, the issue of durability of composite based structures needs to be clarified. Composite parts have a high potential to be damage tolerant structures, due to their intrinsic redundancies, but the behaviour of such components is inherently complex, mainly due to the presence of heterogeneities at various scales. At the *microscopic* scale fibres and matrices, the basic constituents, exhibit different properties such as stiffness, strength, thermal and hygroscopic swelling. At the *mesoscopic* scale, adjacent plies arranged with different fibre orientations with respect to a reference direction exhibit different hygrothermal behaviour, giving rise to important differences in the response to hygrothermal solicitations. Therefore, the material *heterogeneity* at the micro and meso scales contribute to stresses, so-called hygrothermal stresses.

Moreover, not only *material heterogeneity* has the potential to create stress. Temperature and moisture *gradients* can induce stresses as well, due to the fact that constituents are not free to expand but are constraint by the surrounding media. Therefore, hygrothermal strains are required to fulfil compatibility relations. Those time-varying stresses are currently active under service life where environmental conditions are rarely constant and uniform. Finally,

[1] Centro Italiano Ricerche Aerospaziali (CIRA), Via Maiorise, 81043 Capua (Ce), Italy
[2] Institut de Recherche en Génie Civil et Mécanique (GeM), Université de Nantes, BP 406, 44602 Saint-Nazaire, Cedex, France
[3] Département Mécanique et Matériaux, Centre SMS, Ecole Nationale Supérieure des Mines de Saint Etienne, 158 Cours Fauriel, 42023 Saint-Étienne, Cedex 2, France
E-mail: vautrin@emse.fr

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heterogeneous media, such as composites, sustained stresses under hygrothermal loading due to both their heterogeneous nature *and* temperature and moisture gradients.

In the literature, evidence of damage due to temperature differentials, moisture conditioning and thermal and hygrothermal cycling has been reported ([1–3] for instance). It is obvious that hygrothermal fatigue may promote both intra-laminar and inter-laminar damage, under the form of matrix cracking, matrix-fibre debonding and ply-delamination.

Usually, models neglect the effects of temperature gradients compared to water concentration gradients since the heat diffusion rate is considerably higher than moisture diffusion. Therefore, the hygrothermal and mechanical problems are solved by assuming spatially homogeneous but still time-dependent temperature fields. However, a coupling between the temperature and the moisture diffusion rate should be considered since diffusion is thermally activated according to an Arrhenius's law.[4]

Before investigating the hygrothermal and mechanical problems for plate structures, let us review a classical one-dimensional problem to emphasize the coupling between the average diffusion coefficient and the time period in the case of periodic temperature and moisture conditions. We consider a semi-infinite solid subjected to sinusoidal like varying surface concentration -or temperature- ([5], chapter II). The problem is one-dimensional along x-axis and the internal concentration c_{int} is governed by the following equation:

$$\frac{\partial^2 c_{\text{int}}(x,t)}{\partial x^2} - \frac{1}{D}\frac{\partial c_{\text{int}}(x,t)}{\partial t} = 0 \qquad (1)$$

for $x > 0$ and $t > 0$. D is the diffusion coefficient.

The external concentration c_{ext} which is applied varies according to the following temporal law:

$$c_{\text{ext}} = A\cos\omega t \qquad (2)$$

The solution of equation 1 respecting the boundary condition 2 is:

$$c_{\text{int}}(x,t) = Ae^{-2\pi x/\lambda}\cos\left(\omega t - \sqrt{\frac{\omega}{2D}}x\right) \qquad (3)$$

The wavelength λ is defined by:

$$\lambda = 2\sqrt{\pi D\tau} \qquad (4)$$

τ being the period of the cycle. A property of the solution 3 is that the amplitude of concentration oscillations diminishes like $e^{-2\pi x/\lambda}$. At a distance of one wavelength from the surface, the amplitude of oscillations is reduced by a factor $e^{-2\pi} = 0.0019$. An important conclusion then can be drawn: concentration oscillations reside inside the solid up to a distance e_0 which is practically equal to λ. In the case of external varying environmental conditions: temperature *and* moisture, one may define, by analogy with the example above, a parameter e_0 as follows:

$$e_0 = 2\sqrt{\pi\int_0^\tau D(t)dt} = 2\sqrt{\pi\Delta(\tau)\tau} \qquad (5)$$

e_0, whose definition and physical signification has been stated by,[6] has the same meaning and mathematical structure as λ but takes into account a varying diffusion coefficient D. $\Delta(\tau)$ is the average diffusion coefficient over a period. Equation 5 is, in principle, valid for homogeneous materials, however, for multi-ply or multi-component materials, an equivalent diffusion coefficient can be considered. It has to be noted that e_0 depends on both the material properties, through the coefficient of diffusion, and on the *type* of cycle, through the period τ. In other words, e_0 the thickness of that boundary region, where the fluctuating regime prevails, is related to both material and cycle characteristics and does not vary with the cycle number.

First, the paper introduces the modelling of the moisture concentration field in plates under cyclical conditions. Hygrothermal stresses are then simulated in laminated plates by using CLT. Finally, the paper focuses on the special case of *unsymmetric*

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Table 1.
Hygroscopic properties of the IM7/977-2 carbon-epoxy material.

A(eq.10) (mm^2/hrs)	B(eq.10) (K^{-1})	C(eq.9)(%)	b(eq.9)
7.18	−2910.2	0.0007	1.6036

plates. The capability of the model to simulate practical situations -from simple to complex- in the case of cyclical hygro-thermal loading is shown. The aim of this work is clearly to contribute to the modelling and simulation of hygrothermally stressed composite structures. The method here presented is simple, based on straight hypotheses and therefore ideal for design purposes. Table 1 and Table 2

Moisture Concentration

In mathematical terms the one-dimensional problem along the z direction for a plate structure is as follows. Given a spatial region plate of thickness e composed by n *sub-plies* of thickness e_i bounded by two parallel planes $0 < z_i < e_i$, the field equation reads:

$$\frac{\partial c_i(z_i t)}{\partial t} = D_i(t)\frac{\partial^2 c_i(z_i,t)}{\partial z_1^2} \quad \forall \quad 0 < z_i < e_i, \quad t > 0 \quad \forall i, \quad i = 1,\ldots,n \tag{6}$$

with the following boundary and interface conditions:

$$\begin{gathered} c_i(z_i,0) = 0 \\ c_i(e_i,t) = \alpha_{i,i+1} c_{i+1}(0,t) \quad \forall i, \quad i = 1,\ldots,n-1 \\ D_i(t)\frac{\partial c_i(e_i,t)}{\partial z_i} = D_{i+1}(t)\frac{\partial c_{i+1}(0,t)}{\partial z_{i+1}} \quad \forall i, \quad i = 1,\ldots,n-1 \\ c_1(0,t) = c_a(t) \\ c_n(e_n,t) = c_b(t) \end{gathered} \tag{7}$$

$D_i(t)$ is the diffusion coefficient of the i^{th} sub-region and depends on temperature according to a relation of the type:

$$D_i(t) = A_i exp\left(\frac{B_i}{T(t)}\right) \tag{8}$$

with A_i and B_i constants. $\alpha_{i,i+1}$ is a constant which indicates the ratio between the saturation level of the i^{th} sub-region and that of the $(i+1)^{th}$ one. In fact, equilibrium conditions impose, at the interfaces between different materials, equality of the solute chemical potential, not of their concentration. Temperature and surface concentrations are periodic functions of time of period τ. Actually, temperature can be assumed to be uniform through the thickness of the body.

The problem 6–7 can be solved by numerical methods, finite difference method for instance. However an analytical solution (*transient regime*) can be approached by using average quantities. The methodology was proposed by[7] for laminated cylinders and here is applied to laminated plates. The method consists in reducing the original problem 6–7 to an *equivalent* problem with *constant* diffusivity and *constant* external concentrations.[8]

In a slender composite plate, as mentioned, the diffusion process can be assumed essentially one-dimensional: however the plate can be considered also *homogeneous* with respect to the diffusion process, provided their plies are all made with the same material.

Then, by introducing *average* quantities, such as the *average coefficient of diffusion*, $\Delta(\tau)$, and the *average external concentration* $\hat{c}_\infty$:

$$\Delta(\tau) = \int_0^t D(t)dt, \qquad \hat{c}_\infty = \frac{1}{\Delta(\tau)} \int_0^t D(t)c_\infty(t)dt, \tag{9}$$

Table 2.
Mechanical properties of the IM7/977-2 carbon-epoxy material.

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	α_1 ($^\circ C^{-1}$)	α_2 ($^\circ C^{-1}$)	β_1	β_2
152	8.4	4.2	0.35	$0.09 \cdot 10^{-6}$	$28.8 \cdot 10^{-6}$	0	0.6

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the concentration inside the plate assumes the following form if the concentration at $t=0$ is $c_i=0$:

$$c(z, N-k) = \hat{c}_\infty - \frac{4\hat{c}_\infty \pi}{\beta^2} \sum_{n=0}^{\infty} \sum_{i-0}^{k-1} \left((-1)^n (2n+1) exp\left(-\frac{(2n+1)^2 \pi^2 (N-i)}{\beta^2} \right) \right) \times \cos\left(\frac{(2n+1)\pi z}{e} \right) \forall \quad (-e/2) < z < (e/2) \tag{10}$$

with:

$$\beta = \sqrt{\frac{e^2}{\Delta(\tau)}} \tag{11}$$

N is the number of cycles for saturation, equation 10 gives the concentration field after $N-k$ cycles. Finally, the complete through-thickness solution is composed of the previous transient regime (*analytical solution*) and a fluctuating regime (*numerical solution* obtained by finite difference) close to the surfaces (Figure 1).

Hygrothermal Stresses

The hygrothermal field is believed to be responsible for free strains $\mathbf{E}^{HT}$. Then the hypothesis of linear elastic behaviour can be formulated, admitting that the stress is only function of the actual composition c and the actual strain $\mathbf{E}$, independently of the path chosen to reach this state, one may consider the following paths:

a change of composition from $c=0$ to $c(>0)$, at zero stress, producing a strain $\mathbf{E}^{HT}$,

a change of stress from $\mathbf{S}=\mathbf{0}$ to $\mathbf{S}$ at constant c.

In this case, one obtains:

$$\mathbf{S} = \mathbb{C} : (\mathbf{E} - \mathbf{E}^{HT}) \tag{12}$$

where $\mathbf{S}$ is the stress, $\mathbb{C}$ is the stiffness tensor and $\mathbf{E}$ is the mechanical strain.

The stiffness tensor depends on concentration and temperature in the general case. Hygrothermal free strains are given by the classical following relation:

$$\mathbf{E}^{HT} = \alpha \Delta T + \beta \Delta m \tag{13}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are anisotropic tensors of thermal and moisture expansion, while ΔT and Δm in this case represent temperature and composition *variations* with respect to a reference condition.

The stress growth due to hygrothermal fields is here a reversible process and can be set to zero at a certain value of concentration and temperature (so-called *stress free* state). The stress assessment needs the assumption of *a stress free state* which is

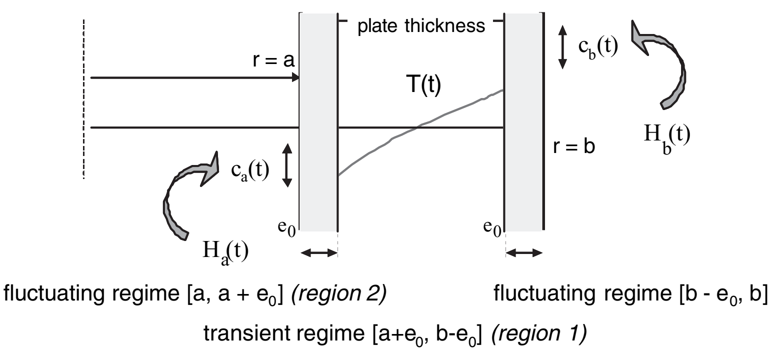

Figure 1.
Fluctuating and transient regimes through the plate thickness.

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mainly dictated by experimental observation.

Within the context of the CLT, the elastic constitutive equation (eq. 12) is substituted by:

$$\mathbf{S} = \mathbb{Q} : (\mathbf{E} - \mathbf{E}^{HT}) \tag{14}$$

where $\mathbb{Q}$ represents the classical *plane stress* stiffness tensor, $\mathbb{Q}$ The axes of orthotropy of any ply are usually rotated with respect to the reference frame (x, y, z) attached to the plate.

By assuming Kirchhoff hypothesis and defining the classical stress and moment resultants, one gets the following laminate constitutive equation:

$$\begin{pmatrix} \mathbf{N} \\ \mathbf{M} \end{pmatrix} = \begin{pmatrix} \mathbb{A} & \mathbb{B} \\ \mathbb{B} & \mathbb{D} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{E}^0 \\ \mathbf{K} \end{pmatrix} - \begin{pmatrix} \mathbf{N}^{HT} \\ \mathbf{M}^{HT} \end{pmatrix} \tag{15}$$

Solving a hygrothermal problem within the framework of the classical lamination theory implies generally the following steps:

calculation of the appropriate homogenised quantities $\mathbb{A}$, $\mathbb{B}$ and $\mathbb{D}$ tensors and hygrothermal resultant force and moment,
calculation of the total strains $\mathbf{E}^0$ and curvatures $\mathbf{K}$,
calculation of ply residual stresses through the material stress-strain relations.

Usually, the free hygrothermal strains are not compatible with the kinematics imposed by the Kirchhoff hypothesis, which postulates linear variation of strains over the thickness of the plate. In principle, displacements should not be assigned *a priori* and should be derived by equilibrium *and* compatibility conditions. A way to circumvent the problem has been proposed by[9] who suggested to approximate hygrothermal fields by *piecewise* linear functions:

$$c(z) = a_k z + b_k \tag{16}$$

where:

$$a_k = \frac{c_k - c_{k-1}}{e_k}$$
$$b_k = c_{k-1} - \frac{c_k - c_{k-1}}{e_k} z_{k-1} \tag{17}$$

The actual hygrothermal fields are discretized over spatial regions called *sub-plies*, with thickness e_k (Figure 2). One sub-ply may be representative of a true physical ply or of a part of it. In this latter case, sub-plies do not have an immediate physical sense but are introduced for discretization purpose only. According to the discretized hygrothermal fields 16 and 17, resultant hygrothermal forces and moments reduce to simple summations over the M sub-plies.

A large class of laminates deform even under uniform hygrothermal fields alone, as these fields produce both thermal forces *and* moments. This behaviour is due to the ply arrangement and is reported as *thermo-elastic coupling*. A typical example is given by laminates of the 0/90 type.

For composite plates subjected to pure hygrothermal loads and *free to deform* a classical plate theory *without transverse shear strain* can be employed. By integrat-

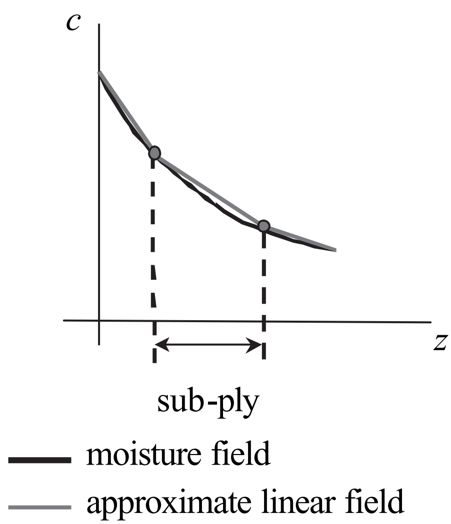

Figure 2.
Piecewise linear discretization of the true hygrothermal field over "sub-plies.

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ing this simple plate problem, one finds the following out-of-plane displacement field for a general laminate:

$$w(x,y) = -\frac{1}{2}(ax^2 + by^2 + cxy) \tag{18}$$

This is the exact solution for the *linear* plate problem. a, b and c are integration constants, which in the linear case correspond to the plate curvatures in x, y axes. For a 0/90 unsymmetric plate $c = 0$, that is twisting does not occur. a and b depend on the material properties and linearly on the temperature differential. If $e_0 = e_{90}$ then $a = -b$: in other words, curvature are of equal magnitude and opposite sign, the deformed shape is a saddle.

It can be observed experimentally that the deformed shape of cured 0/90 unsymmetric plates with $e_0 = e_{90}$ and with certain in-plane dimensions *is not* a saddle.[10–12] In place of the saddle shape (Figure 3a), two cylinders, along the x (Figure 3b) or the y (Figure 3c) direction, exist: one cylinder can be snapped into another by applying an external force.

The phenomenon was explained, for the analogous case of isotropic plates subjected to edge forces, by,[13] who simulated it taking into account geometrical *nonlinearities*, that is, nonlinear relationship between strains and displacements. For composite plates under hygrothermal loads an analogous approach can be undertaken.

Usually plates and shells are so flexible that moderate/large rotations can develop even with small deformations. This allows to use a model based on a simplified nonlinear strain tensor due to Von Karman.[14] In that case, strains take into account the moderate/large rotations but remain in the elastic range, so that no permanent failure/yield occurs. Starting from the Von-Karman strains, a nonlinear theory of plates, known as the Föppl-Karman theory (see, for instance,[15]), can be developed. The nonlinear Föppl-Karman equations can be solved explicitly in few simple cases: For composite plates with elastic and thermoelastic coupling an explicit analytical solution becomes unpractical or impossible, as the nonlinear equations are strongly coupled. An alternative way of solution consists in solving approximately the variational equations of motion to find equilibrium configurations.

For equilibrium configurations, the total potential energy E_T must be a local extremum. Locally stable (i.e., in the presence of relatively small static/dynamic

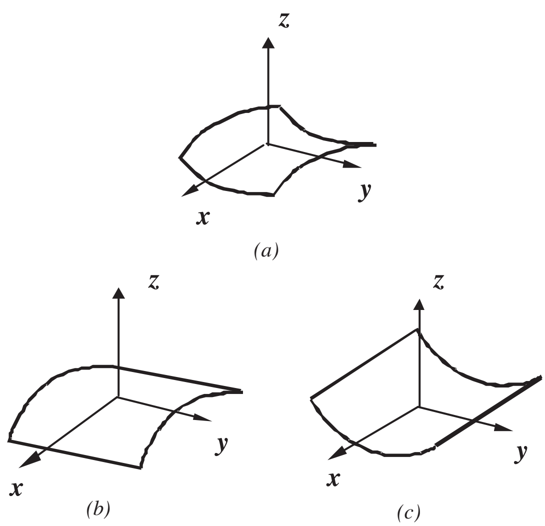

Figure 3.
Cured shape of 0/90 unsymmetric composite shapes.

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perturbations) configurations are those for which the total potential energy is a minimum. The solution is classically approximated through Rayleigh-Ritz, Galerkin or Finite Element procedures. The Rayleigh-Ritz method consists in searching approximate displacement solutions, satisfying kinematics boundary conditions only.

In the search for the right out-of-plane displacement functions one may start, as first proposed by,[10] by using those which come from the linear theory, equation 18. Depending on the material properties, the size, the stacking sequence, the hygrothermal load, equation 18, although looking very simple, allows several deformed shapes. For example one may find saddle shape solutions ($a = -b$, $c = 0$), quasi-saddle solutions ($a \neq -b$, $c = 0$), cylindrical shapes along the x direction ($a > 0$, $b \approx 0$, $c = 0$), twisted shapes ($a = b = 0$, $c = 0$) and so on. For 0/90 plates, the influence of the in-plane shear strain is expected to be negligible, thus the parameter c can be assumed equal to zero.

For a composite plate under hygrothermal loads, the total potential energy is defined as:

$$E_T = \int_V \left(\frac{1}{2}\mathbf{E} : \mathbb{Q} : \mathbf{E} - \alpha T : \mathbb{Q} : \mathbf{E} - \beta m : \mathbb{Q} : \mathbf{E} \right) dV \tag{19}$$

In the presence of transient hygrothermal fields, the modified CLT model discussed can be applied and the last two integrals in relation 19 can be replaced, with good approximation, by summations over M sub-plies.

Identification of Hygrothermal Behaviour

The modified CLT model, introduced in the previous section, is able to simulate the behaviour of thin plates ($(L_x, L_y)/e > 10$) even in the presence of ''rough'' moisture profiles and steep gradients.[14] In the present section, the model is applied to an industrial case, which concerns supersonic flight. Part of the present results have been already presented in a paper by.[16]

Actually, supersonic flight presents some specific features which are particularly aggressive for a composite material. The aerodynamic friction at a Mach number higher than one produces rapid heating of the frame structures and the external temperature may reach relatively high values. Temperature and moisture fluctuations may then produce severe stress fluctuations and gradients.

The *supersonic* cycling consists of a time period at constant environmental conditions, simulating the aircraft maintenance, followed by a periodic repetition of m temperature and humidity cycles. The aircraft maintenance consists of three months at 23 °C and 50% *RH*, after which a so-called *humid state* is established. The *humid state* does not need *a priori* to be a saturated state. One temperature-moisture cycle is detailed in Figure 4.

The cycle can be roughly divided in five parts:

the *take-off* phase (point **A**), at ambient conditions (as for the maintenance),
the *subsonic* phase (point **B**), characterized by low temperatures, –50 °C, and very low external relative humidity assumed to be equal to 0 % *RH*,
the *supersonic* phase (point **C**), characterized by high temperatures, 130 °C, and low relative humidity (as for point B),
the *landing* phase (point **D**), where a cycle ends.

The m *supersonic* cycles are repeated until a *pseudo-dry* saturated state is reached. Thus m practically is supposed to coincide with N, the number of cycles needed for saturation. Simulations of the hygrothermal fields are performed by employing the analytical solutions and the finite difference method. The finite difference solution is needed for several reasons: first, it is important to check the analytical solution against the numerical one. Then, a numerical solution is necessary for the first *supersonic* cycles: for $N - k < 10$ the series

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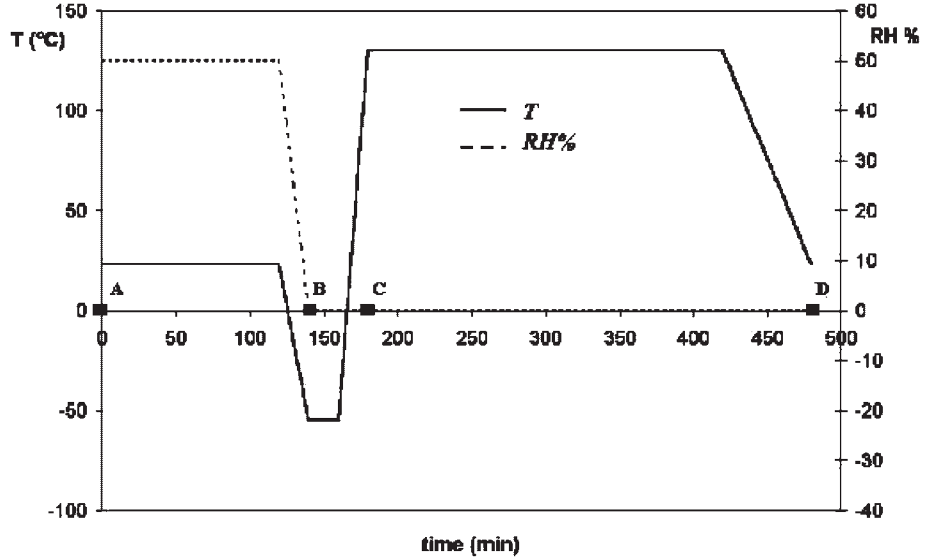

Figure 4.
Detail of the supersonic temperature-moisture cycles.

in relation 10 converge very slowly and an analytical solution becomes unpractical. Finally, the finite difference solution gives the discretized moisture field adopted by the modified CLT model.

The evolution of the through-thickness moisture fields versus the number of temperature-moisture cycles can then be simulated and the drying process of the material due to the supersonic flight at temperatures close to 130 °C has been clearly proved.[16]

Internal service stresses are related to the calculated moisture profiles, however they depend strongly on the manufacturing process. In other words, it is essential to determine *the initial state of stress*, which is due to the cure process. The cure induced stresses are here calculated by assuming a uniform temperature differential $\Delta T = T_{amb} - T_{cure}$. The *stress free* state is characterised by *a stress free temperature* T_{sf}, which is not necessarily equal to T_{cure}. Actually, an accurate knowledge of the true temperature differential $\Delta T = T_{amb} - T_{sf}$ should automatically take into account the effect of non-thermoelastic sources of residual stress, thus providing a reasonable prediction of the corresponding total stress. In the present case, the temperature differential was taken equal to $\Delta T = -187\,°C$.

In Figure 5, the evolution of the *a* and *b* curvatures is presented for a square plate as a function of the temperature, starting from the *stress-free temperature*, at which the plate is assumed to be flat. The classical lamination theory gives the paths A-B, in

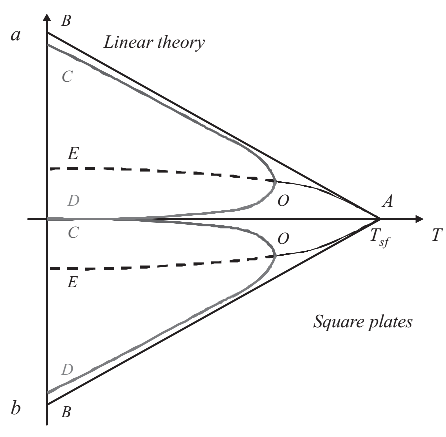

Figure 5.
Schematic behaviour of 0/90 plates under thermal stress.

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which the a and b parameters are, for each value of temperature, of equal magnitude and opposite sign (saddle shape). The non-linear theory provides saddle-shaped deformed solutions until point O, at which a bifurcation phenomenon occurs. In fact, the saddle shapes become unstable (path O-E), while quasi-cylindrical deformed shapes become available on stable post-bifurcation branches. For example, path O-C represents a deformed configuration with $a > 0$ and b which tends to zero increasing the temperature differential.

The deformed shape is progressively *more cylindrical* along the x direction moving from high to low temperatures. Due to symmetry, a cylindrical deformed shape may develop along the y direction (path O-D). The two deformed shapes have, in theory, the same energetic content, thus they may be both exist at a certain temperature. The appearance of one shape rather than the other is driven by the presence of imperfections. Analogous behaviour is found for laminated plates subjected to general hygrothermal conditioning, moisture absorption tends to counteract the effects observed in Figure 5. Therefore, for instance, conditioning the plate while keeping the temperature constant at a value less than T_O (the temperature at point O), that is, beyond the bifurcation point, when the plate develops cylindrical deformed shapes, may promote again transition to saddle deformed shapes. Those plate deformations have been experimentally assessed by utilizing a fringe projection system which delivers the out-of-plane displacement field of the plate surface at each test temperature.[12]

In Figure 6 a transient moisture absorption condition is now analysed. The adimensional *average* composition m and the adimensional parameter $a^* = aL^2/e, b^* = bL^2/e$ are pictured as functions of the adimensional time $t^* = sqrt\ (Dt/e^2)$ for several values of adimensional length L^* $(L_x = L_y = L)$. The initial state $t^* = 0$ coincides with the *after curing* state. Initial deflections (a^*) are issued from residual curing stresses. The figure represents a typical absorption test, with m increasing from zero to saturation. As t^* increases the parameter a^* decreases, due to the moisture absorption that tends to counteract the effects of curing. For $L^* = 7.5$ the initial shape is a saddle, this shape is kept all over the transient state. Analogous considera-

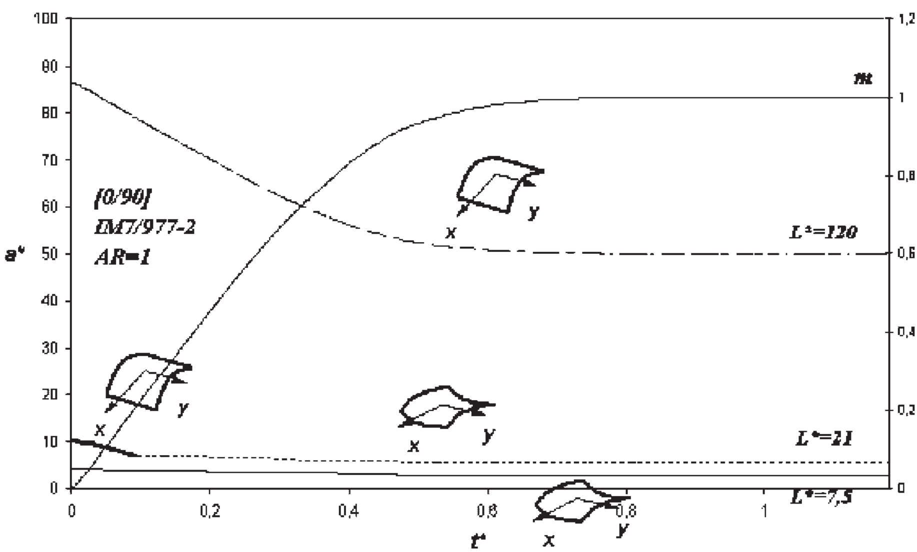

Figure 6.
Transient evolution of the a^* parameter during an absorption test for $L^* = 7.5$, 21, 120 (Material: IM7/977-2).

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tions hold for $L^* = 120$, for which the initial and transient shapes are cylindrical (along the x or y directions indifferently). On the contrary, for $L^* = 21$ the plate has initially a cylindrical shape, which is then converted to a saddle at a certain time and for a certain critical value of m, indicated by m_{cr}. The behaviour of plates with $L^* = 21$ is detailed in Figure 7. It should be noted from figures that both the time and the in-plane dimensions play a major role on the response of the plate. For example, during the first times of the absorption test, the curvature decrease of plates with $L^* = 21$ (the out-of-plane displacement decrease) is faster than that of plates with $L^* = 7.5$ or $L^* = 120$.

Finally, in Figure 8 the evolution of the average moisture composition (m %) as a function of the *supersonic* hygrothermal cycles is presented, in order to relate the hygrothermal evolution to the changes in shape or deflection during cycles.

The first phase of the conditioning (from dry to *humid* state) is characterized by classical Fickian absorption behaviour, which varies linearly with sqrt(t). Now $t^* = sqrt\ (\Delta(\tau)t/e^2)$ As already mentioned, the *supersonic* hygrothermal cycling promotes drying up to a *pseudo-dry* state: however during cycles, hygrothermal fluctuations due to varying external conditions are produced. It can be noted that the composition decrease is "bounded" by two curves which represent envelope curves of the start (point **A**) and the end (point **D**) of each cycle. These fluctuations are almost negligible at the beginning of the cycling process and increases with increasing cycles. Again the response to cycling of 0/90 plates is influenced by their geometrical arrangement (in-plane dimensions, thickness).

Conclusion

The present paper shows the capability of a classical, hygrothermoelastic model to easily simulate complex, transient and cyclical, environmental conditions and predict both hygrothermal stress tensor and structural deformations. The structures studied here are flat plates which are considered sufficiently slender to apply one-dimensional conduction-diffusion theories and *plane stress* constitutive laws. For those plates which deform under the effect

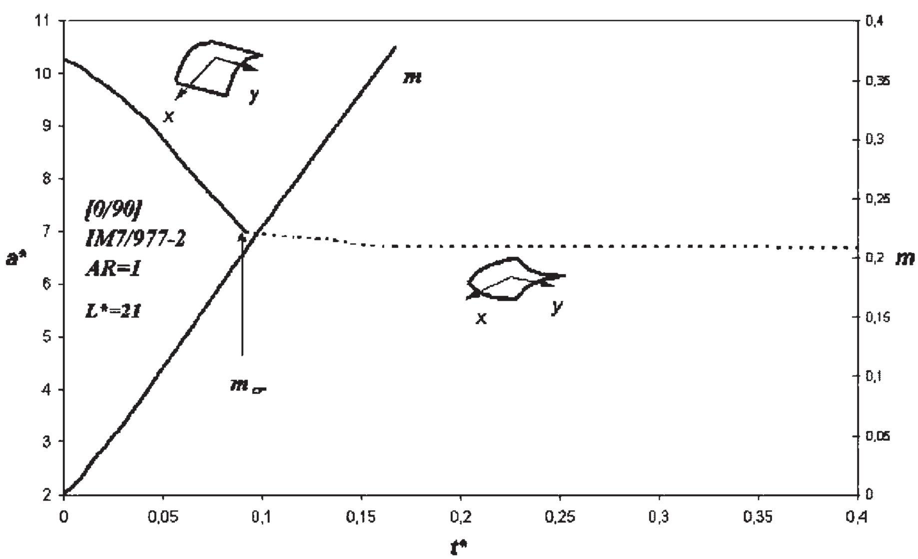

Figure 7.
Transient evolution of the a^* parameter during an absorption test for $L^* = 21$ (Material: IM7/977-2).

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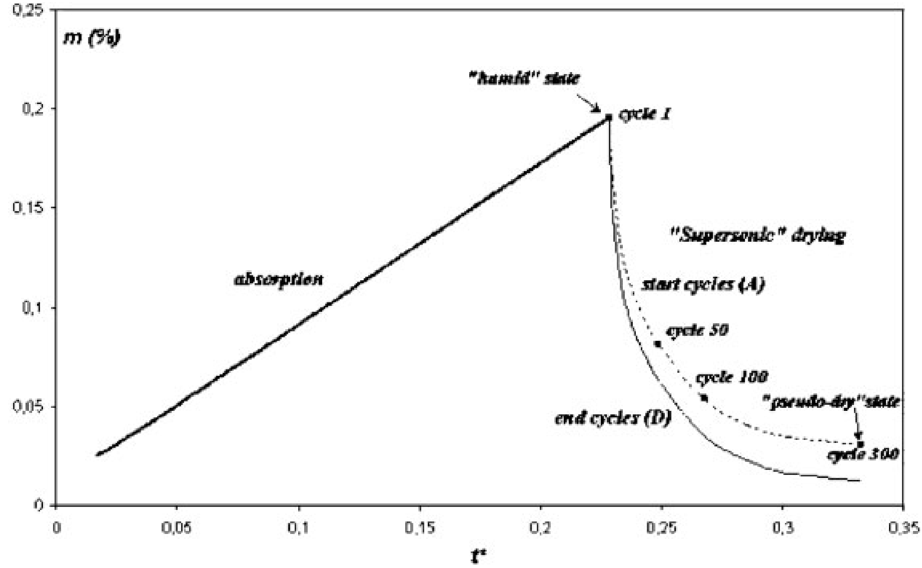

Figure 8.
Transient evolution of hygroscopic *average* composition *m* during the supersonic hygrothermal cycling (Material: IM7/977-2).

of hygrothermal fields, a large displacement - moderate rotations model has to be taken into account.

When hygrothermal cycling is considered, the thickness e_0 of the zone subjected to a fluctuating regime –and where fatigue damage is likely to occur- depends on both the material coefficient of diffusion and period of the cycles. In particular, the higher the period, the thicker this zone.

Laminated plates, employed for experimental purposes, can be *tailored* through the model to lead to more refined and reliable material data. In this way, the hygrothermoelastic stresses and the pertinence of the model itself can be appreciated experimentally and used to build up new routes to identify material behaviour under more realistic loads.

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